Modeling a Supercapacitor using PLECS®

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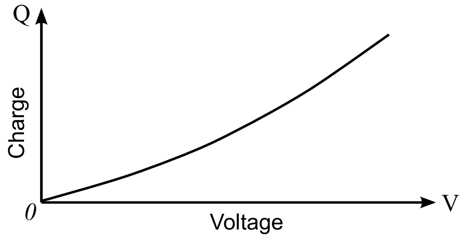
1 Introduction

Due to their high capacitance and low impedance, supercapacitors are well-suited for energy buffer applications that demand a large storage capacitance or a high pulse current capability. In fuel-cell, wind turbine or backup generator applications, the large storage capacitance of the supercapacitor is utilized to meet the power shortfall during start-up and transient operation.

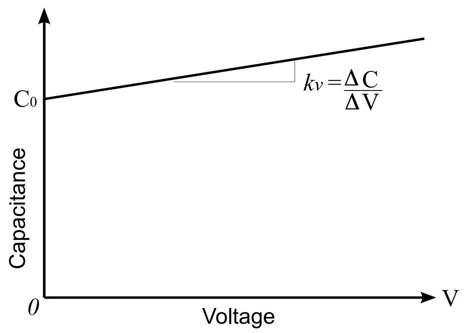
For pulsed power applications, the low internal impedance of the supercapacitor is exploited. For example, a supercapacitor can be connected in parallel with a battery in a hybrid electric vehicle to enhance the pulsed power ability of this higher impedance supply. The supercapacitor supplies or absorbs the large current pulses that occur during engine starting or regenerative braking, improving the transient response and efficiency of the battery supply.

In this report, two supercapacitor models are presented. A simplified model that represents the supercapacitor as a voltage-dependent capacitor with a static internal resistance is first detailed. For transient simulations where frequency-dependent effects are significant, the model is extended to account for short-term self-discharge effects and variations in internal resistance.

The implementation of the supercapacitor models using PLECS is described, and the small-signal impedance or the frequency-dependent model is calculated to depict the effective internal resistance and capacitance during transient operation. Lastly, a combined electrical-thermal supercapacitor model that models temperature rise due to internal power dissipation is given.



(a) Charge vs. voltage.



(b) Capacitance vs. voltage.

Figure 1: Non-linear charge and capacitance characteristics of a supercapacitor.

2 Background

2.1 Supercapacitor construction

Supercapacitors, also referred to as ultracapacitors or double-layer capacitors, achieve a very high energy density with a special wound foil construction. A thin layer of highly porous carbon particles known as activated carbon lies on each foil in order to create a large surface area, and an electrolyte solution is absorbed into the carbon layers [1]. The large surface area and extremely small separation distance results in a very large capacitance value. However, the small separation distance between the electrodes results in a low breakdown voltage of approximately 3V. Therefore, in practical applications, multiple supercapacitors are connected in series to achieve a useful output voltage.

2.2 Effect of voltage

The internal structure of the supercapacitor is affected by an increased accumulation of charge. As the charge and voltage increase, the effective dielectric constant increases. One possible explanation for this is that the applied voltage extends into the carbon electrode causing a space charge, or an electronic capacitance component to develop [2]. The net result is that a supercapacitor exhibits a non-linear charge-voltage characteristic such as that depicted in Fig. 1(a).

Since the capacitance is defined as

$$C = \frac{Q}{V}, \tag{1}$$

the capacitance characteristic is also non-linear. A typical capacitance characteristic of a supercapacitor is depicted in Fig. 1(b). At zero voltage, the supercapacitor has a base capacitance, C_0, and as the voltage increases, the capacitance increases in an approximately linear fashion. The capacitance can be therefore modeled as a function of voltage using:

$$C(v) = C_0 + k_v v \tag{2}$$

2.3 Frequency-dependent effects

A double-layer supercapacitor does not behave as an ideal capacitor. The construction of the supercapacitor introduces non-ideal effects that affect the impedance of the supercapacitor. The typical frequency-dependent resistance characteristic of a supercapacitor is shown in Fig. 2.

At low frequencies below 0.01 Hz, the leakage effect dominates. The leakage effect is caused by current flowing through the separator, internal charge redistribution and self-discharge. Self-discharge, or leakage, is caused by two mechanisms: diffusion of excess ionic charges at the electrodes and impurities in the supercapacitor materials. This results in a DC leakage current and a discharge characteristic with a short- and long-term time constant. Typically, the short-term time constant is in the order of seconds and the long-term time constant is in the order of minutes.

From $0.01 - 10$ Hz, the DC resistance of the supercapacitor, caused by the ionic resistance of the electrolyte, becomes the dominating factor. Above this frequency, the electrolyte behaves as a conductor and the effective resistance between the supercapacitor terminals is the electrical resistance of the contacts and electrodes. This is also referred to as the AC resistance, R_{ac}. Stray inductance in the order of nH causes an increase in the internal impedance above 1 kHz.

3 Supercapacitor Models

Two supercapacitor models are presented in this report: a simplified model that represents the supercapacitor as a voltage-dependent capacitance and a frequency-dependent model that includes leakage effects and internal resistance variations in addition to the capacitance.

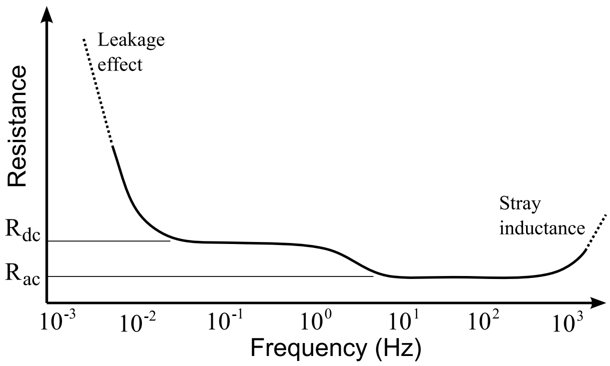


Figure 2: Typical frequency-dependent resistance of a supercapacitor.

3.1 Simplified model

A supercapacitor can be modeled in simplified form as an RC network, with an internal DC resistance and a linear voltage-dependent capacitance. This model is suitable for applications where the energy stored in the capacitor is of primary importance and the transient response can be neglected. Shown in Fig. 3, the simplified model uses a PLECS Variable Capacitor component model to implement the voltage-dependent capacitance. The variable capacitor model is based on the equation:

$$\begin{aligned} i &= \frac{d}{dt}(C \cdot v) \\ &= C\frac{dv}{dt} + v\frac{dC}{dt} \end{aligned} \tag{3}$$

Since the voltage and its derivative are calculated internally by the solver, the capacitance, C, and its derivative with respect to time, dC/dt, are required as external inputs.

Since dC/dt cannot be calculated easily, Eq. (3) is rearranged using the chain rule to allow the second term to be expressed as a factor of ${}^{dv}/_{dt}$ rather than ${}^{dC}/_{dt}$:

$$\begin{aligned} i &= C\frac{dv}{dt} + v\frac{dC}{dv}\frac{dv}{dt} \\ &= \frac{dv}{dt}\left(C + v\frac{dC}{dv}\right) + 0 \\ &= \frac{dv}{dt}C_1 \end{aligned} \tag{4}$$

This rearrangement allows the variable capacitance characteristic shown in Fig. 1 to be implemented by supplying the term C_1 to the capacitance value input and setting the capacitance derivative input to zero. Calculating $C + v\frac{dC}{dv}$ for Eq. (2) yields $C_1 = C_0 + 2k_v v$.

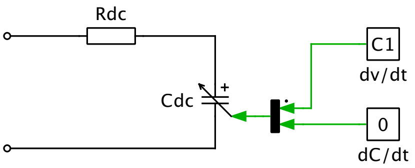


Figure 3: Simplified supercapacitor model with internal DC resistance and linear voltage-dependent capacitance, $C(v) = C_0 + k_v v$. The capacitor is modeled in PLECS by supplying the term $C_0 + 2k_v v$ to the capacitance value input.

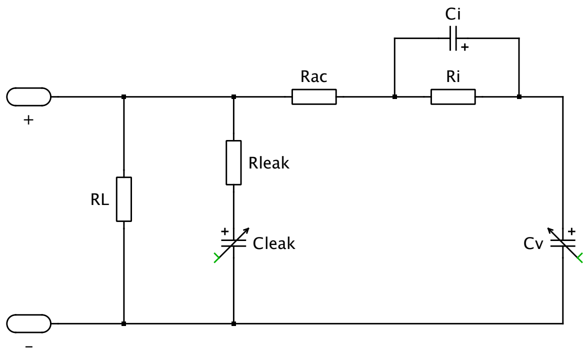


Figure 4: Lumped parameter supercapacitor model.

3.2 Frequency-dependent model

In applications where the transient response or self-discharge behavior must be simulated, the supercapacitor model must include frequency-dependent phenomena such as AC resistance and charge redistribution. Different frequency-dependent models have been presented in literature, and the most common type is based on a three-stage RC network such as those described in [3, 4]. These models are parameterized based on experimental measurements and do not distinguish between the AC and DC components of the series resistance. In addition, the model presented in [4] neglects leakage effects.

A lumped parameter model that accounts for the frequency-dependent impedance in addition to the voltage-dependent capacitance is shown in Fig. 4. The advantage of this model is that it accounts for frequency-dependent phenomena down to the slowest leakage effect, making it useful for transient simulations. The model parameters can be based on experimental tests or known approximations that relate to typical double-layer supercapacitors.

The lumped parameter model is customizable by entering the input parameters, which are described below:

- V_{dc} - rated voltage of capacitor.
- C_{dc} - capacitance at rated voltage.
- k_c - voltage-dependent capacitance (F/V).
- R_{dc} - DC resistance.
- R_{ac} - AC resistance.
- f_{ac} - AC resistance crossover frequency.
- I_L - leakage current.
- r_{cleak} - leakage capacitance as a ratio of total DC capacitance.
- t_{leak} - time constant of short-term leakage effect.

The voltage-dependent capacitance is modeled in a similar manner to the simplified supercapacitor model. However, the voltage-dependent capacitance component is divided between the main storage capacitance, C_v, and the leakage capacitance C_{leak}. The capacitance values as a function of voltage are:

$$C_{leak}(v) = k_{leak} V_{dc}$$

$$C_v(v) = C_0 + k_v V_{dc}$$

where

$$k_{leak} = \frac{C_{dc}\, r_{cleak}}{V_{dc}}$$

$$C_0 = C_{dc} - k_c V_{dc}$$

$$k_v = k_c - k_{leak}$$

The short-term self-discharge behavior is modeled with the RC combination, R_{leak} and C_{leak}, and the leakage resistance is calculated from the given time constant:

$$R_{leak} = \frac{t_{leak}}{C_{leak}}$$

The transition between DC and AC resistance is modeled with the components R_{ac}, R_i and C_i, where R_i is the difference between the AC and DC resistance and capacitor C_i sets the crossover frequency between the AC and DC resistance:

$$R_i = R_{dc} - R_{ac}$$

$$C_i = \frac{1}{2\pi f_{ac} R_{ac}}$$

For high frequency input voltages, C_i acts as a short circuit and the effective series resistance is R_{ac}. Below f_{ac}, resistor R_i adds to the AC resistance.

The resistor R_L accounts for the DC leakage current and is calculated as follows:

$$R_L = \frac{V_{dc}}{I_L}$$

This resistor, however, can be neglected because its impact is negligible over the short-term time range.

It should be noted that most manufacturers do not provide sufficient parameters to determine the frequency-dependent effects. Parameters such as the voltage-dependent capacitance and leakage time constant should ideally be determined from experimental results. In many cases, approximations can be used as a starting point. The following approximations were obtained from experimental results presented in literature [3, 2, 5]: $R_{ac} = 0.5R_{dc}$, $kc = 0.1C_{dc}$, $f_{ac} = 1$ Hz, $t_{leak} = 10 - 100$ s.

3.3 Simulation results

An example 2600 F capacitor was created using the frequency-dependent lumped parameter approach and the simplified approach. The parameters for the lumped parameter model are given in Appendix C. A ± 30 A current pulse was applied to the supercapacitor models, and the output voltage responses are shown in Fig. 5. The leakage effect, which causes a decay in voltage after the initial current pulse, can be observed with the lumped parameter model.

To obtain the small-signal frequency response of the supercapacitor, the state space matrices at the operating point are first extracted using the MATLAB *linmod* function. The frequency response is then obtained using the *plbode* function, which determines the small-signal resistance and capacitance from the real and imaginary components of the frequency response.

A Bode plot of the small signal resistance and capacitance is depicted in Fig. 6. The crossover between DC and AC resistance can be seen to occur at a frequency of 5 Hz and at this frequency, the small-signal capacitance is close to zero. At low frequencies, the small-signal capacitance is approximately 3000 F, which is higher than the stationary capacitance of 2600 F. This simulation result corresponds very well with experimental results presented for an identical supercapacitor in [2].

In order to explain the difference between the stationary and small-signal capacitance, the small-signal capacitance is analytically derived from Eq. (2) as follows:

$$C = \frac{Q}{v} = C_0 + k_v v \tag{5}$$

$$Q = C_0 v + k_v v^2 \tag{6}$$

$$C_{diff} = \frac{dQ}{dv} = C_0 + 2k_v \tag{7}$$

The small-signal capacitance, C_{diff}, is therefore larger than the stationary capacitance in Eq. (2) by a factor of $k_v v$. It should be noted that C_{diff} is equal to the term C_1 in Eq. (4).

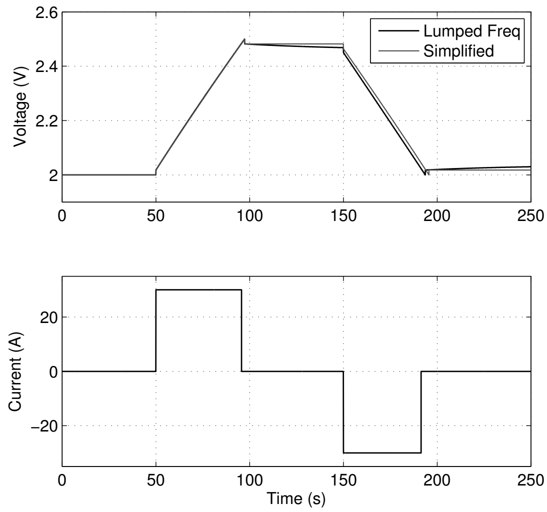


Figure 5: Voltage response of lumped parameter and simplified models of a 2600 F supercapacitor during a ± 30 A current pulse. The leakage effect is observed with the lumped parameter model.

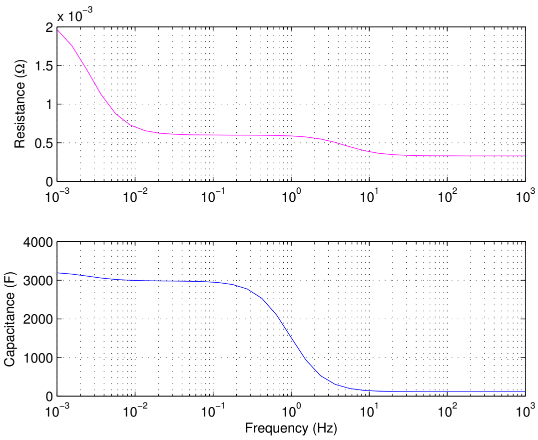


Figure 6: Small-signal resistance and capacitance of a 2600 F lumped parameter supercapacitor model at an operating voltage of 2.5 V.

4 Combined Electrical-Thermal Model

Using the PLECS Heat Sink component, it is possible to add thermal properties to the supercapacitor model in order to examine the internal losses and consequent heating of a supercapacitor during pulsed operation. An example thermal model, *Supercap_thermal.mdl / Supercap_thermal.plecs,* is provided to demonstrate this technique. The supercapacitor is placed on a heat sink, which automatically absorbs all resistive losses and acts as a source for the thermal circuit. This model assumes the internal resistance values themselves do not change due to the internal heating. Above 0°C, this is a reasonable assumption [5].

5 Conclusion

In this report, a simplified supercapacitor model and a frequency-dependent supercapacitor, modeled using a lumped parameter circuit were presented. The lumped-parameter model was also extended to account for thermal behavior. The lumped parameter model is useful for short-term transient simulations up to the time frame of tens of seconds, since it differentiates between AC and DC resistance, and models the short-term self-leakage effect. The long-term leakage effect is ignored because the model is not intended to simulate long rest periods. The lumped parameter model can be easily customized in order to model supercapacitors of different sizes and characteristics.

Appendices

A Simulation Files - PLECS Blockset

Example files used for simulating different supercapacitor models using PLECS Blockset accompany this application note:

- *Supercap_1500F.mdl*: A 1500F supercapacitor model appropriate for transient simulations where frequency-dependent effects are significant. This model accounts for short-term self-discharge effects and variations in internal resistance.
- *Supercap_2600F.mdl*: A 2600F supercapacitor model appropriate for transient simulations where frequency-dependent effects are significant. This model accounts for short-term self-discharge effects and variations in internal resistance.
- *Supercap_2600F_simplified.mdl*: A simplified 2600F model that represents the supercapacitor as a voltage-dependent capacitor with a static internal resistance.
- *Supercap_2600F_thermal.mdl*: A combined electrical-thermal 2600F supercapacitor model that models temperature rise due to internal power dissipation.
- *Supercap_2600F_bode.mdl*: A 2600F supercapacitor model with special Simulink ports added for frequency analysis using the *bode_supercap.m* function.
- *bode_supercap.m*: A function that will generate a Bode plot for the *Supercap_2600F_bode.mdl* model as well as plots for the small-signal resistance and capacitance vs. frequency.
- *plbode.m*: A helper function for generating Bode plots that can be modified by the user for a specific model.

B Simulation Files - PLECS Standalone

Example files used for simulating different supercapacitor models using PLECS Standalone accompany this application note:

- *Supercap_1500F.plecs*: A 1500F supercapacitor model appropriate for transient simulations where frequency-dependent effects are significant. This model accounts for short-term self-discharge effects and variations in internal resistance.
- *Supercap_2600F.plecs*: A 2600F supercapacitor model appropriate for transient simulations where frequency-dependent effects are significant. This model accounts for short-term self-discharge effects and variations in internal resistance.
- *Supercap_2600F_simplified.plecs*: A simplified 2600F model that represents the supercapacitor as a voltage-dependent capacitor with a static internal resistance.
- *Supercap_2600F_thermal.plecs*: A combined electrical-thermal 2600F supercapacitor model that models temperature rise due to internal power dissipation.

C Supercapacitor Parameters

Example parameters used to represent lumped capacitor models of a 2600 F and a 1500 F that are commercially available today are shown in Table 1.

Table 1: Parameters of example supercapacitors.

DC capacitance	C_{dc}	2600 F	1500F
Rated DC voltage	V_{dc}	2.5 V	2.5V
Voltage-dependent capacitance	k_c	250 F/V	150F/V
DC resistance	R_{dc}	0.6 mΩ	1 mΩ
AC resistance	R_{ac}	0.33 mΩ	0.47 mΩ
AC crossover freq.	f_{ac}	5 Hz	10 Hz
Leakage current	I_L	5 mA	3 mA
Leakage capacitance factor	k_l	1/20	1/20
Leakage time constant	t_{leak}	33 s	33 s

References

[1] "Super charged," *IEEE Spectrum*, vol. 42, pp. 32–37, January 2005.

[2] F. Rafik, H. Gualous, R. Gallay, A. Crausaz, and A. Berthon, "Frequency, thermal and voltage supercapacitor characterization and modeling," *Journal of Power Sources*, vol. 165, pp. 928–934, March 2007.

[3] S. Noh, J. Choi, H. Kim, and E. Lee, "Psim based electric modeling of supercapacitors for line voltage regulation of electric train system," in *IEEE International Conference on Power and Energy*, pp. 855–859, 2008.

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[5] R. Kotz, M. Hahn, and R. Gallay, "Temperature behavior and impedance fundamentals of supercapacitors," *Journal of Power Sources*, vol. 154, pp. 550–555, March 2006.